An introduction to theory of time scales

Short course

The theory of time scales is very recent and it was introduced by Stefan Hilgher in his PhD thesis in 1988 (see [3]). This theory unifies continuous and discrete analysis and it extends those theories to the cases "in between". This short course consists of a series of five modules, which are described below.

1. **Introduction to time scales and delta derivative.** In this module, we introduce the jump operators, the graininess function, and the delta derivative for a function defined on a time scale. Also, we present important properties such as product rule and the quotient rule, discuss the derivatives of polynomials and of rational functions, as well as we present a series of examples of time scales that arise from applications.

2. **Integration theory on time scales.** In this module, we present the definition of regulated, rd–continuous, and predifferentiable functions, and presents the existence theorem for antiderivatives. Also, we present the definition of the delta integral and its properties. Improper integrals are discussed and several examples are given. Time scales polynomials are introduced and Taylor's theorem is presented.

3. **Linear dynamic equations and the time scales exponential function.** This module introduces the regressive group and defines the exponential function on time scales. Many properties and examples of this exponential function are given. Also, we show how the exponential function is used to derive variation of parameter formulas. The circle dot multiplication is introduced and the resulting regressive vector space is discussed.

4. **The Laplace transform on time scales.** In this module, we present the unification (and extension) of the famous Laplace transform (for the continuous case) and Z-transform (for the discrete case), as well as we discuss the general Laplace transform for a general time scale, presenting many of its properties. We also present a table for Laplace transforms and give several examples. Also, the convolution theorem is discussed.

5. **Applications.** In this module, we present some applications of dynamic equations on time scales for population models and economic problems.

The main references for this short course are [1, 2].

References

[1] M. Bohner and A. Peterson, *Dynamic Equations on Time Scales: An Introduction with Applications*, Birkhäuser, Boston, 2001.

[2] M. Bohner and A. Peterson, *Advances in Dynamic Equations on Time Scales*, Birkhäuser, Boston, 2003.

[3] S. Hilger, *Ein Maβkettenkalkül mit Anwendung auf Zentrumsmanningfaltigkeiten*, PhD thesis, Universität Würzburg, 1988.